The Journal of Entrepreneurial Finance

Volume 1
Issue 3 *Spring 1992*

Article 4

December 1992

U.K. Small Firm Bankruptcy Prediction: A Logit Analysis of Financial Trend-, Industry-, and Macro-Effects

Robert C. Cressy
SME Centre, University of Warwick

U.K. Small Firm Bankruptcy Prediction: A Logit Analysis of Financial Trend-, Industry-, and Macro-Effects

Robert C. Cressy

I. INTRODUCTION

Much work has been done in the last two decades to estimate the determinants of bankruptcy of large firms. (See Storey, Keasey, Watson and Wynarczyck [19]—henceforth SKWW—for references). Very little apart from the early work of Edmister [7] in the United States and more recently the work of SKWW [19] in the United Kingdom appears to have been attempted in the area of small firm bankruptcy. This paper goes some way to remedy the deficiency by estimating conditional logistic probability regressions for small firm bankruptcy on a recently constructed U.K. accounts database.

Our methodology like Edmister [7] and SKWW [19] highlights the importance of financial ratio trend effects on small firm bankruptcy potential. However our empirical specification of trends is more general than that of previous work. We show that a five-year lag structure in financial ratios generates the best model for the data. In addition we examine the influence of industry effects and explore the use of year-dummies proxying economy-wide influences on small firm bankruptcy potential. These latter can be thought of as proxies for macroeconomic effects, the precise nature of which are the subject of further research.

Our results thus demonstrate quite conclusively that several years' data on financial ratio variables are required to provide reasonable predictive accuracy on small firm solvency rather than the one year's accounts information traditionally used in large firm analysis.

The paper also demonstrates by drawing on the work of Palepu [15] and Zmijewski [23] the empirical biases that can arise from uncritical use of state-based sampling techniques.

The paper is organized as follows. Section II provides a summary of the recent critical literature in the econometric analysis of bankruptcy. There we set up the basic framework for use in Section III which reruns the SKWW

Robert C. Cressy • SME Centre, University of Warwick, Coventry, England.

The Journal of Small Business Finance, 1(3): 233-253
ISSN: 1057-2287

analysis of U.K. small-firm bankruptcy. Section IV introduces some novel elements into the empirical approach. Using an expanded variable set we examine the impact of industry- , year- and financial trend-effects on small firm bankruptcies. Section V summarizes the results and draws some policy conclusions.

II. FOUNDATIONS OF BANKRUPTCY ANALYSIS: THE RECENT CRITIQUE

The explanation of why firms fail and the prediction of their failure has been a subject of interest to academics in finance for the last two decades. The earliest statistical analysis of the causes of bankruptcy is Beaver [4] who used univariate accounting ratio techniques to attempt to provide an "early-warning system" for impending financial disaster. However it was Altman [1] who first introduced multivariate techniques into the literature by way of MDA (Multivariate Discriminant Analysis) and Altman's 1968 paper has spawned a huge literature on the prediction of bankruptcy. (See, e.g., Zavgren [22] for a survey). Much of this literature involves verbal theorizing to justify the choice of particular financial ratios used as explanatory variables in the statistical model. Recent contributions however have provided formal mathematical models of the bankruptcy process which can be more scientifically tested using ratio data (e.g., Cressy [5]).[1]

Much of the informal statistical work in the area has recently been placed into question by two perceptive papers (Zmijewski [23] and Palepu [15]). These papers constitute a sophisticated critique of the statistical methods that have been employed by drawing on the earlier analysis of Manski and Lerman [14]. The latter identifies the need for appropriate adjustments to traditional model estimation techniques in the context of a theory of state-based sampling. Palepu and Zmijewski however establish the precise statistical effects of erroneous sampling methods on the prediction of bankruptcy using MDA and LIMDEP (Limited Dependent Variable) methods. Finally, the papers contain a discussion of the criteria for optimal cutoff points in bankruptcy classification in the context of a literature that commonly assumes arbitrary values.

To get a flavor of this critique consider a population containing two groups of observations *A*'s and *B*'s. In this context a state-based sample is a non-random sample where the researcher chooses the sample proportions of *A*'s and *B*'s with different probabilities. Sample proportions of *A*'s and *B*'s will not therefore reflect population proportions as would be the case with random sampling. There are often good statistical reasons for choosing state-based rather than random sampling. For example, if one group has only small numbers in the population it may be desirable to select disproportionately in its favor. On the other hand, adjustments to estimation techniques need to be made to avoid a number of serious statistical errors. In particular, over-representation of one group in the sample relative to the population will bias upwards the classification accuracy of that group in any statistical attempt at discrimination within the sample and will yield the misleading impression that the model can predict well in the wider population. This impression is enhanced if the holdout sample used to "verify" the model's predictive power is itself subject to sampling bias similar to that in the estimation sample.

Consider the question of classification cutoff points. A cutoff value in the logistic model serves to divide the population into two groups on the basis of probabilities. If a firm's estimated probability is above the cutoff it is classified as bankrupt and if below as active. Intuitively the choice of cutoff point must reflect the expected gains/losses over alternatives. Suppose we evaluate the losses arising from Type I and Type II errors in classification and imagine starting with an arbitrary probability cutoff value p_0. Firms are classified as bankrupt or active as their observed probability p_i falls above/below p_0. This rule will generate an expected loss with weights the posterior joint probabilities of pairs (p_i, y_i), $y_i = 0$ or 1. If the cutoff value is varied marginally then the Type I/II error will in general rise/fall and the Type II/I error will fall/rise, changing the expected loss accordingly. If the expected loss falls, it follows that we have not chosen p_0 optimally. Since most studies have not considered this issue it is highly probable that their assumed cutoffs are not optimal in the above sense.

More precisely, Palepu [15] has shown that state-based sampling methods in the absence of the appropriate adjustments lead to (i) upward bias and inconsistency in estimates of the probability of bankruptcy, and (ii) upward bias to classification accuracy. Arbitrary cutoff points for classification are also shown to lead to erroneous results, though the assessment will depend on the precise model of bankruptcy assumed. Zmijewski [23] demonstrates these points empirically on U.S. large firm data and also addresses an additional "missing data problem." This relates to the fact—potentially very significant for small firms—that some firms may have one or more model variables missing in a given period. Usually the solution in the literature has been simply to ignore such observations. Zmijewski shows that this is a valid procedure only under very special circumstances. Failure to deal with the problem is shown to generate sample selection bias with again potentially serious effects on prediction.[2]

Palepu [15] provides the appropriate Bayesian formulae to summarize the issues and will be useful in explaining the empirical methods to be adopted below.

The sample probability of firm i becoming bankrupt in a given period is given by

$$\begin{aligned} p_B' &= \Pr\{i \in B \mid i \in S\} \\ &= \frac{\Pr\{i \in B, i \in S\}}{\Pr\{i \in S\}} \\ &= \frac{\Pr\{i \in S \mid i \in B\}}{\Pr\{i \in S \mid i \in B\}\Pr\{i \in B\} + \Pr\{i \in S \mid i \in A\}\Pr\{i \in A\}} \\ &= \frac{\pi_B\, p_B}{\pi_A\, p_A + \pi_B\, p_B} \end{aligned} \tag{1}$$

where

A, B and S are sets of active, bankrupt and sampled firms $\pi_A = n_A/N_A$ and $\pi_B = n_B/N_B$;
N_A, N_B are the population numbers of A's and B's, respectively;
n_A, n_B are the sample numbers of A's and B's, respectively;
$p_B = \Pr\{i \in B\}$ is the population probability of being a B.

Note that if $\pi_A = \pi_B$ (random sampling) then $p_B' = p_B$, i.e., the population and sample probabilities coincide. For state-based or non-random sampling $p_B' \neq p_B$. In the case most commonly encountered in the literature, that of equal sampling, $n_A = n_B$. Since N_B is usually much smaller than N_A we have $p_B' > p_B$, i.e., an upward bias is imparted to the bankruptcy probability by the sampling method adopted. Palepu shows that the optimal cutoff probability under plausible assumptions[3] is given by the following equality

$$f(p \mid i \in B) = f(p \mid i \in A) \tag{2}$$

where $f(p \mid i \in Z)$ is the conditional probability density of observing a predicted bankruptcy probability p if the firm is actually a Z ($= B$ or A). This is equivalent in fact to choosing the cutoff point so as to minimize the sample χ^2 of the model and under certain assumptions to minimize the expected loss from misclassification.

The population probability of bankruptcy of firm i in a conditional logit model is given by

$$p_B = (1 + \exp(-\beta' x_i))^{-1} \tag{3}$$

where x_i is a vector of financial ratios for firm i and β a vector of parameters. Using equations (1) above (3) yields

$$p_B = [1 + (\pi_A/\pi_B) \exp(-\beta' x_i)]^{-1} \tag{4}$$

where we recall that π_A, π_B are the sample proportions of A's and B's, respectively. This formula can be rewritten as

$$p_B = [1 + \exp(-(\gamma + \beta' x_i))]^{-1} \tag{5}$$

where $\gamma = \ln(\pi_B/\pi_A)$. Thus the parameter bias effect of state-based sampling operates entirely through the intercept term. (See also Maddala [13], p. 90). An advantage of this fact is that estimation of probabilities can proceed by maximum likelihood on the state-based sample and the appropriate adjustments to estimated probabilities can be made ex post. However, this method has the drawback that it does not produce maximized likelihood statistics to check goodness-of-fit, a statistic which has then to be estimated by other means (e.g., graphical). The alternative is non-standard MLE using the functional form (5) directly. This is the method employed in the present study.

Optimal cutoff points for classification in the two-state case can be calculated using the Savage regret function.[4] Denoting the estimated probability of bankruptcy by p_i, the set of firms classified into the bankrupt category by $P^* = \{p_i: p_i \geq p^*\}$ and the loss from action a_i when state s_j holds by $L(a_i, s_j)$ the expected loss from the classification scheme is given by

$$\begin{aligned} EL = {} & L(p_i \in P^*, i \in A) P\{p_i \in P^*, i \in A\} \\ & + L(p_i \in -P^*, i \in B) P\{p_i \in -P^*, i \in B\} \\ = {} & L_{BA} \int_{p^*}^{1} f(p \mid y = 0) dp \cdot q_0 + L_{AB} \int_{0}^{p^*} f(p \mid y = 1) dp \cdot q_1 \end{aligned} \tag{6}$$

where L_{BA}, L_{AB} is abbreviated notation for misclassification losses and q_0, q_1 are prior probabilities of being an A or a B, respectively. Minimizing this expected loss or regret with respect to p^* we get

$$\frac{f(p^* \mid y = 0)}{f'(p^* \mid y = 1)} = \frac{L_{AB}\, q_1}{L_{BA}\, q_0}$$

subject to

$$\frac{f'(p^* \mid y = 0)}{f'(p^* \mid y = 1)} = \frac{L_{AB}\, q_1}{L_{BA}\, q_0}$$

Thus at the optimum the marginal expected losses from misclassification based on posterior densities must be equal.[5]

It is worth noting finally before moving on to the empirical analysis that the traditional use of classification accuracy as an assessment criterion usually relies simply on the proportion of successes and failures correctly classified. Since this ignores the associated Type I and Type II error costs discussed above it is extremely misleading: models with large and small prediction error rates are rated as equivalent on this criterion. A much more satisfactory measure of goodness-of-fit and empirical indicator of prediction accuracy is the pseudo-R^2 (see Maddala [13]).[6] This is the primary measure of the explanatory power of the model that we shall use in the empirical work that follows.

III. THE EMPIRICAL LITERATURE ON SMALL-FIRM BANKRUPTCY

Very little work has been done in the area of predicting small firm bankruptcy. Edmister [7] on U.S. data and SKWW [19] on U.K. data are the major contributions in the area. More specific topics are dealt with in Keasey and Watson [10, 11].

Edmister (1972)

Edmister used discriminant analysis in the vein of Altman to examine the bankruptcy characteristics of 42 U.S. small firms. Two analyses were performed, one using firms for whom one year's financial data was available prior to the event date and another using firms for whom three years' financial data was available. Sampling rates were 100% of the bankrupts in each subsample and 15% and 19% of the actives for the one- and three-year sets respectively. He employed a stepwise approach to obviate problems of multicollinearity. Edmister nonetheless found (despite the obvious upward boost to classification accuracy implied by his sample selection procedures) that intertemporal instability displayed by small firm financial ratios meant that the one-year's data model proved useless for bankruptcy prediction into the holdout sample. He concluded that one should use three-year averages instead of annual data and the three-year model so structured predicted well in the holdout.[7]

By way of criticism of Edmister's study we note that apart from the estimation bias implied by Edmister's sample selection procedure the averaging method used in the three-year analyses has the econometric disadvantage that it imposes unnecessary parameter restrictions on the model being estimated. Thus using a three-year average assumes that the regression weights attached to each year's variable value are the same. A general lag structure in which the weights are allowed to be determined by the data is clearly a better specification.

SKWW (1987)

SKWW [19] in a pioneering study of U.K. small firm bankruptcy used discriminant and logit analysis to examine the bankruptcy potential of a set of 636 small U.K. firms. Our discussion of their extensive study will relate primarily to their logit analysis, the technique employed in the present paper.

The SKWW sample was defined as all firms in the Northern region of England with at least one years' published (Companies House) accounts data and with less than 200 employees. In addition, firms had to be limited, single plant, independent, manufacturing companies. The 8 variables used in logit analysis were selected from a larger set of 12 variables by factor analysis to represent the characteristics of liquidity, profitability, and so on thought to be relevant to the prediction of bankruptcy. The set of 12 variables was as follows:

W1. Current assets/Current liabilities
W2. Net profit/Total assets
W3. Fixed assets/Total assets
W4. (Pre-tax profit + depreciation)/Total debt
W5. (Pre-tax profit before directors' fees + interest)/Total debt
W6. (Total debt excl. bank overdraft)/Total assets
W7. Current assets/Total assets[8]
W8. (Current assets-stock)/Total assets
W9. Net profit/Fixed assets
W10. Fixed assets/Net worth
W11. Net profit/(Current assets-current liabilities)
W12. Pre-tax profit/Net worth

IV. THE PRESENT STUDY

Our empirical study falls into two main parts. First, we re-estimate SKWW's equations on the same database and examine their statistical credentials. Second, we respecify the model introducing a general lag structure and additional variables, and estimate this for comparison. A novel element in our approach in addition to the specification of trends is the incorporation of macroeconomic variables represented by year-dummies.[9]

A Rerun of SKWW (1987)

To simplify terminology in what follows we make the following definitions.

Definition 1: The regression of a dependent variable y observed in period t on a set of independent variables $x_1, \ldots x_n$ observed in periods $t-k, t-k+1, \ldots t-1$ is called a "$[t-k, \ldots . t-1 \rightarrow t]$ regression."

Definition 2: A complete observation on q variables is one in which each of the q variables has a non-missing value.

Eight variables W1, W2, W4, W5, W7, W9, W11, W12 were selected by SKWW from the above set by means of factor analysis and a series of 5 annual logit regressions ($[t-k \rightarrow t]$, $k = 1, \ldots 5$) on firms with 3 and 7 consecutive years' "complete" (8-variable) data were run.[10] To minimize sampling bias we ran instead $[t-k \rightarrow t]$, $k = 1, \ldots 5$ regressions using pooled complete observations from the period 1970-1980 on the SKWW subset of 8 variables. Thus initially we imposed no requirement for a panel of individual firm data.

We note that a random *pooled* sample should reflect the annual average number of B's to the annual average number of A's in its sample proportions. The SKWW sample consisted of all the B's and an equal number of A's in the period 1970-1980. We have seen above that to avoid parameter bias, adjustments then need to be made to the estimated coefficients. Unfortunately, SKWW do not in fact do this.

Our group sample numbers for the 8-variable regressions were initially the total number of B's in the 11-year sample 1970-1980 and the average annual number of A's for the subset of variables in question ($L(k-1)Wi$, $k = 1, \ldots 5$, where L denotes the lag operator). The regression results are reported in Table 1 including numbers of A's and B's used in each case.

The equations were then re-estimated using equation (5) above to reflect population proportions. Probabilities and classification statistics were recalculated in each case choosing classification cutoff points optimally. Results are presented in Table 2 together with the SKWW results for comparison.[11]

Discussion

We note from Table 1a that the similarity of the SKWW results with our own is not very great. In particular coefficients differ sometimes substantially not only in magnitude but also in sign. This may be explicable of course by reference to the different sampling criteria employed.[12]

Table 1a
Regressions from the 8-variable Set Without Parameter Adjustment (SKWW Counterparts in Square Brackets)[23]

Years Prior	*C*	*W1*	*W2*	*W4*	*W5*	*W7*	*W9*	*W11*	*W12*
1	-2.562**	-0.266	-3.495**	-0.174	0.381	2.233*	-0.087	-0.014	-0.164
	[-1.300	-0.137	0.952	0.269	-0.958	2.219	0.295	-0.011	-0.207]
2	−2.392**	-0.094	-3.817**	-0.021	0.293	2.265**	-0.014	0.014	0.029
	[-2.076	0.141	2.984	0.227	-0.459	-1.923	-0.086	0.095	-0.007]
3	-1.582**	-0.341**	-2.181*	-0.107	-0.429	2.252**	0.093	-0.173*	-0.050
	[-1.397	-0.427	-0.974	0.113	-0.078	-1.860	0.003	-0.012	-0.004]
4	-1.266**	-0.199*	-0.285	-0.178	-0.694	1.942**	0.131	0.000	-0.046
	[-0.839	0.595	1.047	0.034	0.022	-1.727	-0.041	0.012	0.043]
5	-1.531**	-0.041	-0.320	-0.388**	-1.430*	2.188**	0.341	0.009	-0.124
	[0.314	0.320	-0.123	0.031	0.526	-0.799	0.015	0.057	0.140]
6	-1.622**	-0.011	0.177	-0.050	-0.852	1.963*	-0.358	0.033	-0.118
	[--	--	--	--	--	--	--	--	--]

Note: ** denotes significance at 1% and * at 5% level.

Table 1b
Classification Accuracy, Optimal Cutoffs and R^2 for Regressions in Table 1a (SKWW Counterparts in Square Brackets)

Years Prior	*−2log-LR* ($\chi^2 8$ *df*)	*% Correctly Classified* *B*	*A*	*TOT*	*Optimal Cutoff*	MR^2	N_A	N_B
1	16.37	21	97	83	0.33	0.16*	268	57
	[19.97]	[100]	[3]	[94]	[?]	[?]		
2	25.96	56	73	68	0.28	0.11**	258	88
	[42.62]	[28]	[97]	[88]	[?]	[?]		
3	42.16	65	71	69	0.32	0.16**	243	108
	[39.56]	[41]	[92]	[81]	[?]	[?]		
4	24.06	26	91	69	0.44	0.10**	222	113
	[46.29]	[13]	[97]	[73]	[?]	[?]		
5	38.69	64	67	66	0.34	0.17**	199	102
	[26.47]	[7]	[100]	[69]	[?]	[?]		
6	7.69	40	82	68	0.40	0.09*	174	90
	[--	--	--	--	--	--]		

Notes: (i) Our measures of goodness-of-fit are McFadden's R^2 adjusted for degrees of freedom defined as $MR^2 = (\chi^2(p) - 2p)/(-2\log(L_\omega))$ where L_ω is the maximum likelihood function under the Null hypothesis and p the number of parameters estimated in the Alternative hypothesis.
(ii) ** denotes significance at 1% and * at 5% level.

Table 2
Classification Accuracy, Optimal Cutoffs and R^2 for Regressions from the 8-Variable Set With Parameter Adjustments

Years Prior	*−2log-LR* ($\chi^2 8$ *df*)	*% Correctly Classified* *B*	*A*	*TOT*	*Optimal Cutoff*	MR^2	N_A	N_B
1	9.45	67	70	70	0.02	0.02	268	57
2	13.16	65	60	60	0.03	0.02	258	88
3	38.43	85	42	44	0.03	0.04**	243	108
4	20.16	83	34	36	0.04	0.02**	222	113
5	35.70	80	48	49	0.04	0.05**	199	102
6	22.66	73	50	50	0.04	0.04**	174	90

The maximum R^2 for our initial (unadjusted) regressions is very low (maximum 17% for the 5-years' prior regression).[13] Thus the unadjusted SKWW model does not seem to explain the data well.

Overall classification accuracy on the unadjusted model reported in Table 1B is lower than reported by SKWW in conformity with Palepu's [15] results on classification bias.[14] Classification accuracy on the *B*'s for given cutoff will be lower the lower the sample proportion of *B*'s. SKWW's *B*-classification accuracy from Table 1 is, however, largely less than ours despite the smaller proportion of *B*'s in our sample. This is most probably because of the employment of different cutoff optimality criteria. However, since SKWW do not report optimal cutoffs no precise conclusion can be drawn.

Finally the results for the parameter-adjusted regressions in Table 2 demonstrate very clearly the major effects of classification bias. For constant cutoff optimality criteria the R^2 for all the equations are a fraction of their original values and the maximum R^2 declines to 5% (for the 5-years' prior equation). Despite this we see that the proportion of correctly classified bankrupts is still as high as 85%. This result highlights quite dramatically the dubious nature of most classification reporting procedures in the literature. By concentrating on the percentage of actual *B*'s classified correctly they ignore the prediction error rates associated with this accuracy.[15]

A New Approach to the Empirical Analysis

Before moving on to the empirical analysis proper we introduce four new variables into the model on the basis of their popularity in the financial and accounting literature. They are defined as follows:

W13. Cashflow/Total debt
W14. Equity/Total debt
W15. Quick assets/Current liabilities
W16. Creditors/Debtors

Apart from W14 which measures in some sense the financial risk of the firm's activities these additional variables are indicative of the degree of liquidity of the firm's balance sheet.

We now examine the influence of economy- and industry-wide factors and trends in financial ratios on the explanation of bankruptcy potential of U.K. small firms. To be able statistically to pick up economy-wide effects requires the use of models with explanatory data covering several years rather than one. Thus the economy-wide regressions are essentially pooled-data models. Also while there is precedent in the literature for financial trend analysis we have seen above the econometric specification has been that of 3-year averages. It was pointed out earlier that this is nonoptimal and in what follows we employ a general lag structure to analyze trend effects leaving the data to determine weights to be assigned to individual variables.

The sample used in these regressions is the population of firms with complete observations on current and lagged values of W1-W16 defined above. This is of course not a random sample of small firms from the population of small firms at large but may be thought of as a random sample of firms with (up to) six years' complete data on sixteen variables.[16] Again while it is likely that there is bias in the dataset used due to the substantial propensity of small firms to nonsubmission of accounts [11], the SKWW database unfortunately does not allow estimation of sample selection bias from this source.[17] Finally we note that a disadvantage of imposing the requirement of six years' data to be available for each firm is that the sample size for bankrupts is reduced to 17[18]. The model selection method employed was a forward stepwise procedure using a 10% variable entry/exit criterion.[19]

Industry Effects

The 636 firms in the database fall into 89 industrial (MLH) categories. To make the industry analysis manageable we reduced this number by aggregation to 27 categories. Initial regressions were run on the set of 27 industrial dummies. The results were as follows.

A regression on all 27 variables rejected H_0 at less than 0.5% level but explained only 18% of the variation of probabilities across firms. Furthermore, collinearity among regressors resulted in "only" 14 of the industry dummies being individually significant at the 10% level. Finally, the highest (absolute) partial correlation[20] of any significant independent variable with the dependent variable was only 12%.

We conclude that industry factors have some importance in the explanation of bankruptcies of small firms. However to examine how far the industry dummies' explanatory power persists when the effects of financial ratios have been taken into account we ran an additional regression with financial and industry variables simultaneously included. The results are discussed later.

Year Effects

The fact that observations are pooled in our analysis allows us to examine the differential impact of time-specific or more intuitively of temporal economy-wide effects on bankruptcy probabilities. Macro-effects were examined here by defining year-dummies for each of the years 1970-1980.[21] A regression on these dummies alone was then run. The outcome was as follows.

The model R^2 at 23% was higher than that for the industry model and H_0 was rejected at below the 0.5% level. Likewise the Null hypothesis on individual year dummies was rejected at 7% or below for 1970 and 1976-1980 inclusive. The maximum partial correlation of any year dummy with y was 19% for 1980 followed by 14% for 1978. The partials for the remaining years were much lower.

In conclusion the year- or macro-effects alone explain a significant proportion of the variation of bankruptcy probabilities across firms with some years much more important than others. There are therefore grounds for including year-effects in a more general model with financial ratio trends and to investigate the nature of the specific macroeconomic variables at work.[22]

Financial-Ratio Trend Effects

To examine the effects of trends in financial ratio variables we first ran the regression $[t - 6, \dots , t - 1 \rightarrow t]$ using variables 1-16. The results are presented in Table 3 and we now summarize these.

The R^2 for the financial ratios model alone is 58.1% and significant at below the 0.5% level. Five of the seven variables in the optimal model are significant at 1% or below. Individual partials vary between 11% and 38%. Seventy-one percent of failures are identified at relatively small cost in terms of Type I errors. The failure prediction accuracy of the model at 50% is satisfactory.

Table 3
16-Variable Financial Trend Regression[24]

	C	$L4W2$	$W2$	$L5W1$	$L2W2$	$L5W5$	$L3W12$	$L2W3$
Coeff	-6.7	-.97	-.98	.66	-1.0	-.70	-.79	-2.5
Sig Lvl(%)	.7	.07	.00	.00	.00	3.0	9.4	.00

$N = 993$ $N_A = 976$ $N_B = 17$

$MR^2 = 0.581$ (Sig Lvl(%) = .00) Optimal cutoff = 0.16

% Classified correctly			% Correct Predictions[25]		
B	A	T	B	A	T
71	99	98	50	99	98

Combined Effects Models

The same financial variables are included as in the financial effects model first with industry- and then with year-dummies. The results are presented in Tables 4 and 5. Finally a regression including all three effects was made. The results of this are presented in Table 6.

The optimal models for financial effects (Table 3) financial + industry effects (Table 4) and financial + year effects (Table 5) can be compared.

Financial ratio trend effects are seen to remain important when industry effects have been accounted for (Table 4). Of the industry dummies industries 2, 29 and 25 have coefficients significant at below the 2% level and have partial r's between 14% and 24%. The R^2 for the financial + industry effects model is roughly the same as for the financial model alone at 58.4% and is significant at below the 0.5% level. 59% of failures are identified with a 42% prediction accuracy. Thus by comparison with the financial effects model the financial + industry model has an identical R^2 but a somewhat lower B-classification and B-prediction accuracy. Finally we note that the two models have substantial overlap in the financial variables included.

In conclusion, in terms of statistical significance (both for the regression as a whole and for individual variables) and in terms of explanatory power there is very little to choose between the financial and financial + industry models. Little significance should be placed on the differences in classification accuracy. However, since the industry effects may reflect unrepeatable historical trends in industrial structure, we recommend the financial trends model over the financial + industry model for policy purposes.

Regarding the financial + year model (Table 5) H_0 is rejected at below the 0.5% level. The final model with nine variables explains 57% of the variation of the dependent variable. Eight financial variables appear in the

Table 4
16-Variable Financial + Industry Regression

	C	*L2W9*	*L4W2*	*W2*	*L5W1*	*W3*	*IND2*	*IND29*	*L5W5*	*IND25*
Coeff	-6.7	-1.7	-.90	-.65	.45	-1.6	3.4	1.8	-1.0	3.3
Sig Lvl(%)	1.3	0.0	.05	.07	1.6	.02	.05	1.8	1.1	1.9

$N = 993$ $N_A = 976$ $N_B = 17$

$MR^2 = 0.584$ (Sig Lvl(%) = .00) Optimal cutoff = 0.13

% Classified correctly			% Correct Predictions		
B	*A*	*T*	*B*	*A*	*T*
59	98	98	42	99	98

Table 5
16-Variable Financial + Year Regression

	C	*L4W2*	*W2*	*L5W1*	*L2W2*	*L5W5*	*YR10*	*L2W3*	*L5W7*	*LW2*
Coeff	-7.3	-1.0	-.84	.74	-1.0	-1.2	.93	-3.07	-1.4	-.59
Sig Lvl(%)	.00	.09	.01	.00	.02	.08	.47	.00	.97	2.0

$N = 993$ $N_A = 976$ $N_B = 17$

$MR^2 = 0.570$ (Sig Lvl(%) = .00) Optimal cutoff = 0.42

% Classified correctly			% Correct Predictions		
B	*A*	*T*	*B*	*A*	*T*
47	99	99	80	99	99

Table 6
16-Variable Financial + Industry + Year Regression

	C	*L2W9*	*L4W2*	*W2*	*L5W1*	*W3*	*IND2*	*IND29*	*L5W5*	*IND25*
Coeff	-6.7	-1.7	-.90	-.65	.45	-1.6	3.4	1.8	-1.0	3.3
Sig Lvl(%)	1.3	0.0	.05	.07	1.6	.02	.05	1.8	1.1	1.9

$N = 993$ $N_A = 976$ $N_B = 17$

$MR^2 = 0.584$ (Sig Lvl(%) = .00) Optimal cutoff = 0.13

% Classified correctly			% Correct Predictions		
B	*A*	*T*	*B*	*A*	*T*
59	98	98	42	99	98

Note: Identical results to Table 4.

model all significant at below the 1/2% level except L5W7 and LW2. Partials vary between 18% and 36%. Year effects (Year 6-Year 10) are reduced in the final model to one, Year 10, or 1980. This has a partial correlation of 11% with the dependent variable, but is significant at the .5% level.

Thus the financial + year model explains the data marginally less well than the financial model. The classification accuracy of the model however falls even further below that of the financial effects model to 47% while the prediction accuracy of the model rises above that of the financial model to 80%. Again we are inclined to place little significance on these latter statistics and rely mostly on the R^2. This implies that there is little difference between the models.

This fact is confirmed by the three-effects model. Table 6 is in fact identical to Table 4. Thus by a small margin the best fitting model seems to be the financial + industry trend model. However, there is little to choose between them and for reasons discussed above we still regard the financial trend effects model as the best of the available alternatives.

Interpretation

Profitability (Net profits/Total assets, LiW2) appears in all models (financial, financial + industry and financial + year) and in virtually every prior year as a statistically significant determinant of bankruptcy. It has moreover a quite high negative correlation with the bankruptcy variable (up to 35% for 3 years' prior) and has obvious intuitive appeal. Net profit relative to total debt (LiW5) also consistently appears to reduce the probability of bankruptcy in the early years.

No other variable plays so dominant role as profitability[26] in the small company's financial fortunes. Our results suggest that it should be regarded as the major determinant of bankruptcy for small firms and one for which the influence of trends is paramount.[27] This result is furthermore consistent with financial and economic theories of firm bankruptcy (e.g., Van Horne [20]; Jovanovic [9]).

In addition to profitability, liquidity variables feature importantly in the model. They also display a rather interesting pattern of behavior. In all models we find that liquidity (LiW1, $(\text{LiW3})^{-1}$)[28] is *positively* correlated with bankruptcy. This might be thought to be a spurious correlation since it is usually assumed that more liquid firms are *less* bankruptcy-prone. However, a firm's failure to control costs or overexpansion of sales ("overtrading") may result in a cash flow problem and a reaction of the following form.

As the date of bankruptcy approaches, "doomed" companies attempt to extricate themselves from financial difficulty by expanding their short term loans and overdrafts with the bank).[29] However, this will increase liquidity in the sense of the current ratio (*CA/CL*) only if this ratio was originally less than one and the evidence is that for bankrupt firms other measures of liquidity (e.g., Net Working Capital/Total Assets) are sharply *declining* as the date of bankruptcy approaches.[30] The inability to recognize that evasive action of this kind will not solve the problem can be therefore thought of as a component of the bankruptcy syndrome. The empirical result above then shows that increased borrowing resulting in "improvements" in the liquidity variables in balance sheet, especially at a relatively late stage (e.g., in L2W3) do *not* alter the firm's fundamental problems but rather serve as a signal of the firm's failure to tackle them. Firms that attempt to make such untimely adjustments are identified in the model as *more* (not less) likely to go bust.[31]

Finally the explanatory power these liquidity variables, measured by partial *r*'s is (in the financial model) around 28% so they have some importance in the prediction of insolvency.

It is also of interest to note that the financial economist's darling, the debt-equity ratio, and the accountant's favorite, the quick ratio, play no role in the optimal model. This suggests that they have no *independent* explanatory power in the determination of solvency probabilities. Thus, although a higher debt-equity ratio may increase the likelihood of bankruptcy if this ratio change is merely a *symptom* of some other change in the firm (e.g., declining profitability), then holding the latter constant may anesthetize the gearing effect altogether.

A final comment on the classification accuracy of the financial model is in order. While the number of *B*'s is small both absolutely (a mere 17) and relative to the number of *A*'s (there are 993 of them), 71% are classified accurately. Unlike many models in this area, the cost of this prediction accuracy is not high in terms of Type I errors (*A*'s classified as *B*'s). The ratio of predicted to actual *B*'s is low at 1:1. Comparison with other studies is however impossible because prediction accuracy is generally not reported along with classification accuracy.

V. CONCLUSION AND POLICY IMPLICATIONS

We have shown that it is possible to classify the bankruptcy behavior of small U.K. firms rather well from financial trend data, industry information and macroeconomic effects proxied by year-dummies. The preferred model was one that used simply the financial ratio trend effects as explanatory variables. The model developed is somewhat greedy in its consumption of data, requiring six years' financial statements to generate an R^2 of 58%. Importantly, however, the coefficients of this model are stable and have clear economic/financial interpretations. In particular, the trends of profitability and liquidity variables in the years before bankruptcy occurs constitute a fascinating cautionary tale of the small firm's eventual descent into insolvency.

The policy conclusions for government agencies flowing from the analysis are as follows.[32]

In a population of 1000 firms having six years' financial data, about 20 will become bankrupt in any one year. One-fortieth or about 25 of these 1000 firms should be investigated. They are identified as the set of firms with bankruptcy probabilities in the model (financial trends) above 16% and predicted by the model as insolvent within one year. Within this set of twenty-five firms will be included about 70% of the actual bankrupts in the following year. About 50% or 12 of the identified firms will actually become insolvent within the period.

The precise value of the model to the government agency policy-maker will clearly depend on the costs of investigating 25 firms and the benefits from avoiding the insolvency of 12 such firms. However, the model is undoubtedly the best systematic guide to this kind of policy decision.

The policy conclusions for other categories of investor, e.g., the bank or venture capitalist receiving a loan request from a small company may of course be different. He may not be interested in investigating 25 firms for signs of impending insolvency but will wish to know whether the current loan applicant is likely to go bankrupt. The model has a roughly one in two chance of telling him this correctly and is therefore potentially of considerable use in evaluating his investment prospects.

Directions for Future Research

In a future paper it is intended to examine the underlying macro-effects implicit in the year-dummies used in the present paper. Also in the vein of SKWW [19] and Peel and Peel [16], we intend to examine the effects of adding to the financial trend variables off-balance sheet items such as lags in accounts submission and information on audit qualification. Both the above studies found such measures predictive.

Acknowledgments: I should like to thank the Editor and an anonymous referee of this Journal for helpful comments.

NOTES

1. The advantage of a formal model is that it provides comparative static signs which can be tested.

2. Zmijewski's study finds some bias present from this source. The likelihood of bias is however greater for small firms due to the much higher frequency of missing data. See Keasey & Watson [11] and note 17 (below).
3. Palepu's [15] concern is to provide an economic model of takeovers rather than bankruptcies. However, the methodology has more general applicability. See Press [17, chapter 13].
4. See Savage [18] or Press [17] for details of the general theory.
5. This formula specializes to Palepu's, provided the RHS = 1.
6. While a high R^2 is not a guarantee of predictive ability (witness the position in the presence of multicollinearity) if individual coefficients are highly significant it is in practice a reasonably reliable guide. It is also one used throughout much of the economic/business literature as such in the absence of holdout data. Should such data become available any model should of course be tested to confirm the impression provided by the goodness-of-fit measure. However, chi-squared tests of the ex post fit of the model are much more reliable than the traditionally used classification methods employed in the bankruptcy literature (see Amemiya [2]).
7. It is perhaps worth noting that Edmister is one of the few writers to provide a discussion of the optimal choice of cutoff probability and to examine the effects of chance on classification accuracy.
8. By an accounting identity W3 = 1 − W7. The model is therefore impossible to estimate unless one of these variables is dropped. Since most computer programs do this and the Storey equations do not contain W3 we assume this has been done. The regression package we use automatically deletes redundant variables. We have therefore not deleted one of the two and the results that follow must be interpreted accordingly.
9. To my knowledge no attempt has been made in the literature to explore the effects of macroeconomic variables on individual firm bankruptcies. Wadhwani [21], however, has found that at an aggregate level inflation does (positively) influence bankruptcy rates.
10. We do not repeat precisely these regressions even with parameter adjustments since it is evident *ab initio* that the sampling procedure is nonoptimal.
11. We did not break the total sample up into initial and holdout samples. Since the results on an initial sample are at least as good as on a holdout the results in Table 2 represent an upper bound to the accuracy achievable on a holdout.
12. See note 8.
13. We note however that our results on R^2 are very much of the same order of magnitude as those reported by Palepu [15] for his takeover studies which incorporated appropriate parameter adjustments to allow for sample selection bias.
14. The sample proportion of *B*'s is larger in our study than in the SKWW study even before any parameter adjustment has been made. The effects of this on *overall* classification accuracy are as Palepu shows ambiguous. Thus our results are consistent with Palepu's [15, p. 9].
15. The ratio of predicted *B*'s to actual *B*'s is in this case 19. Thus for every correctly identified *B* there are 19 *A*'s incorrectly classified as *B*'s. Whether this kind of error rate constitutes a problem will of course depend on the decision context and the marginal cost of Type I errors.
16. In fact we use the population of small firms in the database with these characteristics. While we do not pretend that the sample is representative of small firms as a whole it is perhaps more so of the more successful small firms. This feature of the sample is however is interesting in its own right.

17. The extent of bias cannot be estimated à la Heckman [8] since there apparently exists no variable in the SKWW database that is present for *all* observations. The existence of such a variable is necessary to facilitate estimation of the probability of missing financial data for each firm in the estimating sample.
18. The sample proportion of *B*'s is thus around 2%. The sample proportion of *B*'s in the SKWW database over the whole period 1970-1980 (applying to firms of all "ages" of data) is considerably larger at around 10%. The figure of 2% applying here is closer to that for large firms and probably reflects the failure rate of the more "successful" small firms in some general sense of this term.
19. The sense in which the financial models tested are *trend* models is that the *initial* specification includes each variable plus all of its lags. The final model of course includes only those variables satisfying the entry/exit criterion of 10% significance.
20. We define the partial correlation coefficient in this context by $r = [(\text{MLE chi-square} - 2)/(-2L\omega))]^{1/2}$. It lies between -1 and 1 (r is set $= 0$ if MLE < 2) and provides a measure of the contribution of the variables independent of sample size.
21. Since we specify a 5-year lag model in the combined regressions with financial variables, only years 1976-1980 will of necessity be available for estimation.
22. The subject-matter of a future paper will be to identify the precise macroeconomic factors generating these year-effects. Preliminary investigations show that the year-dummies are proxying macro-variables such as inflation rates (seeming to confirm Wadhwani [21]). In the present study such year-dummies have functioned simply as a heuristic device.
23. These results are for the "incomplete" or three consecutive years' financial data firms in the SKWW study. This methodology is the one closest to ours since we do not require consecutiveness at all.
24. To enhance interpretation of the variables of the model we standardized all variables to zero mean and unit variance. This allows regression coefficient ratios to represent marginal rates of substitution between corresponding variables along probability isoquants when evaluated at variable means.
25. Prediction accuracy is defined as the proportion of the original sample *predicted* to be *B*'s (*A*'s) that *are in fact B*'s (*A*'s). It is not "prediction" in the holdout sample sense. It does, however, provide some intuitive guide to the model's ability to predict. Furthermore, as we have noted, many of the studies in this area have used highly biased holdout samples with attendant spurious classification and prediction accuracy.
26. Recall that this definition is effectively of *retained* profit and is different from the economist's and finance theorist's idea of profits. This is because (1) it is not a cash-flow measure (it includes arbitrary allowances for depreciation), and (2) because it does not refer to shareholder distributions. In view of these facts we attempted to test the effectiveness of definitions of profitability closer to the economic concept of cash-flow profits. For example, Net Profit + Depreciation provides a better approximation to operating profit and cash-flow than does the definition used in the text. However, this measure did not perform as well as that presented. We conclude that despite its smaller intuitive appeal Net Profit/Total Assets is the "appropriate" variable to use.
27. It is worth noting that the average rate of profit on total assets for the whole sample is a mere 4%. Thus the typical small firm does not require a catastrophic change of fortunes to slip from solvency into insolvency.
28. Recall that we have not dropped one of the pair W3, W7. Thus the sign of L2W7 is the negative of that of L5W3, which conforms to what is said in the text.
29. This can be seen in a univariate analysis of lagged mean values of (Short Term Loans + Overdrafts)/Total assets. For the set of active firms this is constant at around 10%.

For the set of bankrupts the ratio is steeply increasing (from about 15% to 25%) as the date of bankruptcy approaches. Meanwhile Net Working Capital/TA is steeply declining.

30. I am indebted to an anonymous referee for this point.
31. Presumably the higher bankruptcy-proneness of such firms is not identified by their banks.
32. These points are predicated on the assumption that the goodness-of-fit measures reflect prediction accuracy, and hence on the accuracy of a holdout test.

REFERENCES

[1] Altman, E. I., "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance*, September 1968.

[2] Amemiya, T., "Qualitative Response Models: A Survey," *Journal of Economic Literature*, Volume XIX, December 1981, pp. 1483-1536.

[3] Bathory, A., *The Analysis of Credit*, MacMillan, 1986.

[4] Beaver, W. H., "Financial Ratios as Predictors of Failure," *Journal of Accounting Research*, Supplement, 1966, pp. 71-102.

[5] Cressy, R. C., "Small Firm Bankruptcy as the Bank's Decision Problem," SME Centre Discussion Paper, Warwick Business School, University of Warwick, Coventry, England, 1990.

[6] Edmister, R. O., "Financial Ratios and Credit Scoring for Small Business Loans," *Journal of Commercial Bank Lending*, September 1971.

[7] Edmister, R. O., "An Empirical Test of Financial Ratio Analysis for Small Business Failure Prediction," *Journal of Financial and Quantitative Analysis*, March 1972.

[8] Heckman, J. J., "Sample Selection Bias as Specification Error," *Econometrica*, Volume 47, 1979, pp. 153-161.

[9] Jovanovic, B., "Selection and the Evolution of Industry," *Econometrica*, Volume 50, Number 3, May 1982.

[10] Keasey, K. and R. Watson, "Current Cost Accounting and the Prediction of Small Firm Performance," *Journal of Business Finance and Accounting*, Volume 13, Number 1, Spring 1986, pp. 51-70.

[11] Keasey, K. and R. Watson, "The Non-Submission of Accounts and Small Company Financial Distress," *The Accounting Review*, 1987.

[12] Keasey K., R. Watson and P. Wynarczyck, "The Small Company Audit Qualification: A Preliminary Investigation," *Accounting and Business Research*, Autumn 1988, pp. 324-333.

[13] Manski, C. F. and S. R. Lerman, "The Estimation of Choice Probabilities From Choice-Based Samples," *Econometrica*, Volume 45, Number 8, 1977.

[14] Maddala, G. S., *Limited Dependent and Qualitative Variables in Econometrics*, Cambridge, 1983.

[15] Palepu K., "Predicting Takeover Targets," *Journal of Banking and Finance*, 1986.

[16] Peel, M. J. and D. A. Peel, "Predicting Corporate Failure: Some Results for the UK Corporate Sector," *Omega*, Volume 14, Number 1, 1986, pp. 5-12.

[17] Press, S. J., *Applied Multivariate Analysis*, Holt Rinehart and Winston, 1972.

[18] Savage, L. J., *The Foundations of Statistics*, John Wiley and Sons: New York, NY, 1954.

[19] Storey D. J., K. Keasey, R. Watson and P. Wynarczyck, *The Performance of Small Firms*, Croom Helm: London, 1986.
[20] Van Horne, J. C., "Optimal Initiation of Bankruptcy Proceedings by Debt Holders," *Journal of Finance*, Volume XXXI, Number 3, June 1976, pp. 897-910.
[21] Wadhwani, 1986.
[22] Zavgren, C. V., "The Prediction of Corporate Failure: The State of the Art," *Journal of Accounting Literature*, Volume 2, Spring 1983.
[23] Zmijewski, M., "Methodological Issues Relating to the Estimation of Financial Distress Prediction Models," *Journal of Accounting Research*, Volume 22, 1984, pp. 59-82.